# The Capacity of Multiple Beam Waveguides and Optical Delay Lines

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(Manuscript received July 10, 1968)

*The capacity of a beam waveguide can be increased by transmitting a multitude of Gaussian beams in such a way that they are clearly resolved at the receiving end. Various systems with maximum capacity but different crosstalk sensitivity are discussed. Linking the available channels end to end in an optical cavity produces a delay line or storage device. An optimized system is described which has surprisingly large storage capacity. For the analysis of both lens guides and optical cavities, a phase space representation of Gaussian beams is used which avoids cumbersome mathematics.*

## I. INTRODUCTION

A Fabry-Perot interferometer with curved mirrors can be used as an optical delay line by inserting a laser beam through a small center hole in one mirror.[1] The beam performs many off-axis round trips before leaving the interferometer through the entrance hole.[2] Reference 1 suggests that the injection and retrieval of the beam could be improved by mismatching beam and cavity. A systematic study is carried out here to find the longest folded path that starts and ends in the center hole, thus optimizing the system for maximum storage capacity.

Very similar to this problem is the analysis of a periodic lens guide in which many beams are to be transmitted in such a way that they are clearly resolvable at the receiver end. One such system is a transmission link that forms an image array of modulators in the receiver plane. The possible density of channels is given by the number of resolvable spots in this plane.[3]

The investigation of all possible Gaussian beams transmitted simultaneously in a guide will show that this is only one among many possible systems. All these systems exhibit the maximum theoretical

capacity as given by the classical limit,[4] but may be affected differently by guide imperfections.

The first part of this study will outline a simple geometrical method of describing gaussian beams avoiding the cumbersome mathematics connected with gaussian beam optics.[5] Based on this method, it will be easy to find the optimum storage cavity and to investigate various multiple beam transmission systems.

## II. PHASE PLANE AND PHASE SPACE

In continuous or periodic guiding media, the "phase space" representation of paraxial rays is very convenient. Consider, for example, the two-dimensional continuous lens-like medium in Fig. 1a in which the index of refraction is a function of the transverse coordinate only:

$$n(x) = n_0\left(1 - \frac{1}{2}\frac{x^2}{\Delta^2}\right). \tag{1}$$

Call $\Delta$ the "focusing parameter." The paraxial ray solutions are sine waves with the period

$$P = 2\pi\Delta \tag{2}$$

as shown in Fig. 1b.[6] Figure 1c shows a "phase plane" in which every ray of Fig. 1b is represented by a point. The coordinates of the points

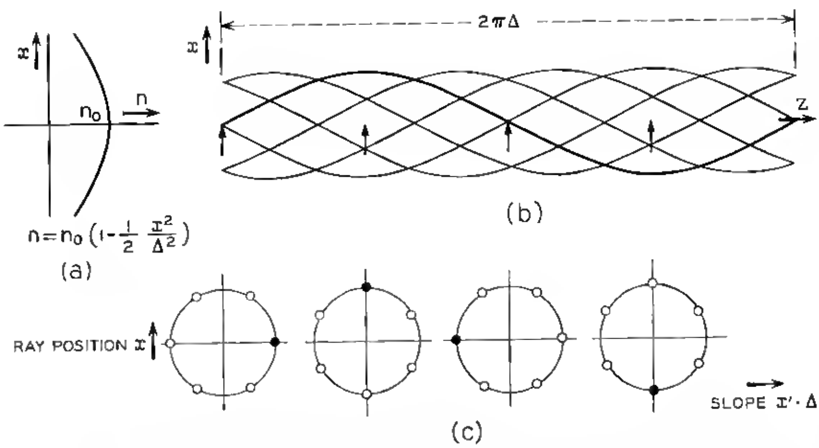


Fig. 1 — Rays in a homogeneous guiding medium. (a) The square-law index profile. (b) Rays oscillating with various phases. (c) The corresponding points in the phase plane.

correspond to the position $x$ and the slope $x'$ of the ray multiplied by $\Delta$. As the rays proceed in the square-law medium, the points orbit around the origin of the phase plane while their position with respect to one another stays the same.

Steier[7] has shown that for every gaussian light beam one can find a packet of rays equivalent to this beam in the sense that the packet envelope gives the beam width and the average ray slope is perpendicular to the beam phase front. The ray packet may be represented by an array of points in the phase plane. Consider, for example, a fundamental gaussian beam propagating in the square-law medium of Fig. 1. The $1/e$-half width of such a beam is

$$w = \left(\frac{\Delta\lambda}{\pi}\right)^{\frac{1}{2}} \tag{3}$$

where $\lambda$ is the optical wavelength. The equivalent ray packet is basically the one shown in Fig. 1b with ray amplitudes $w$. The corresponding points in the phase plane occupy a circle with radius $w$ similar to the presentation in Fig. 1c.

Following these arguments, any gaussian beam—varying in position, slope, or width along the guide—may be represented by its array of points in the phase plane. The points form a "phase spot" in the phase plane whose shape and position determine the beam parameters. Once the phase spot is known at one point along the guide, it can be found for any other point by simply rotating the phase plane. The correspondence rules between the phase spot and the beam parameters follow from Steier's ray racket equivalence and are explained in the following examples.

Figure 2 shows a gaussian beam of width $w$ entering the guide with a slope $\alpha$. The beam phase front is tilted by $\alpha$ and consequently the average slope of all rays in the ray packet must be $\alpha$. This condition is satisfied by a circular phase spot displaced horizontally by $\alpha\Delta$. As the beam proceeds in the guide, the phase spot orbits around the origin of the phase plane. Projection of the phase spot on the vertical axis yields the beam width and position. The horizontal displacement determines the slope. Notice that Fig. 2 and the following figures are two-dimensional beam representations. The phase spots should not be confused with a cross-sectional view of the beam.

Figure 3 shows a beam that enters the guide with a phase front curved with a radius $R$. Consequently, the average slopes of the equivalent rays vary linearly across the ray packet. In the phase plane horizontal

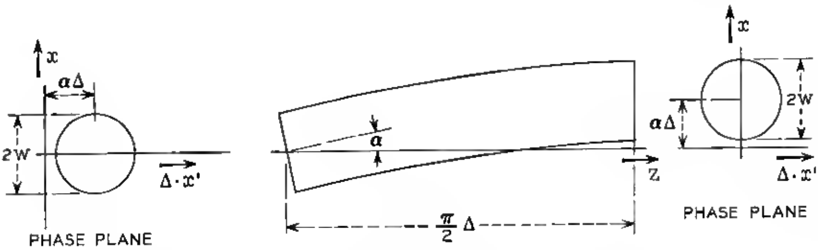


Fig. 2 — The phase spot for a beam entering at an angle $\alpha$.

slices of the phase spot are displaced horizontally by $x\Delta/R$ according to their position $x$ which distorts the circular phase spot to an ellipse. Notice that the area of the phase spot is not changed by this process. From equation (2) one finds that this area is $\pi w^2 = \Delta\lambda$. It is the same for any gaussian beam of a given wavelength in a given guiding medium. A beam, for example, that enters with a plane phase front and a half width $u \neq w$ has an elliptic phase spot with the principal axes $u$ and

$$v = \Delta\lambda/\pi u. \tag{4}$$

If the guiding medium is not homogeneous along the $z$ axis but a periodic sequence of lenses, the phase plane method is still valuable though, with the same convenience, the beam can only be described in the planes of the lenses and not in the sections between. This, however, is in general sufficient because, no matter what the features of tbe gaussian beam, it will always be largest at the lenses and therefore it will be this width that determines the aperture of the whole system.

For a periodic sequence of lenses with focal length $f$, spaced at a distance $d$, the convergence parameter is[6]

$$\Delta = d/\sin\Phi \tag{5}$$

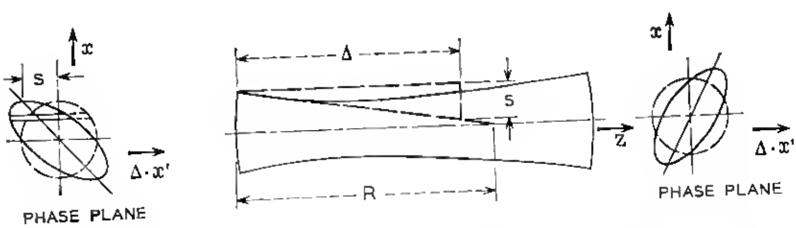


Fig. 3 — The phase spot for a beam entering with a curved phase front.

with

$$\cos \Phi = 1 - d/2f \tag{6}$$

and the equivalent ray period is $2\pi\, d/\Phi$.

For thin biconvex lenses, it is the symmetry planes of the lenses where the Gaussian beam can be defined most conveniently. Beam width and phase front curvature in this plane determine the equivalent phase spot. Counterclockwise rotation of the phase spot by an angle $\Phi$ corresponds to passage from one lens to the next.

The phase plane method may also be extended to nonperiodic structures. It is restricted, however, to the paraxial approximation, to square-law guiding profiles (including uniform dielectrics), and to coherent beams with Gaussian intensity profile and spherical phase fronts.

Notice that the phase plane considers only deflection and displacement in $x$ direction and that a similar definition exists for the $y$ coordinate. The two phase planes combined yield the four-dimensional phase space, and the phase spot becomes a four-dimensional structure.

## III. SPATIALLY INDEPENDENT CHANNELS

The capacity of a beam waveguide can be increased by transmitting several gaussian beams separated spatially. The tolerable crosstalk determines the separation of the individual beams. For convenience, let us describe around every beam a fictitious tube, $k$ times wider than the $1/e$ width, where $k$ is chosen so that the crosstalk requirement is met when these tubes just touch. In practice, the main source of crosstalk will be beam distortion and scattering rather than the spread of the ideal beam. The factor $k$, therefore, will vary from guide to guide according to the tolerances of the guiding components.

Figure 4 shows a two-dimensional square-law medium of width $2a_x$ and the corresponding phase plane. In order for the beams to clear the guide walls, the phase spots must stay within the circle $r = a_x$ while orbiting in the phase plane. Considering that the phase spots require an area $k^2\ \Delta\lambda$ to fulfill the crosstalk conditions, it is easy to find the phase spots that make the best use of the available guide (see Fig. 4). The phase spots determine the beam parameters.

As the beams oscillate in the guide, they overlap in certain areas. There are, however, cross sections spaced by distances $\pi\Delta$ at which all beams are separated. One of these cross sections may be chosen as the receiver plane.

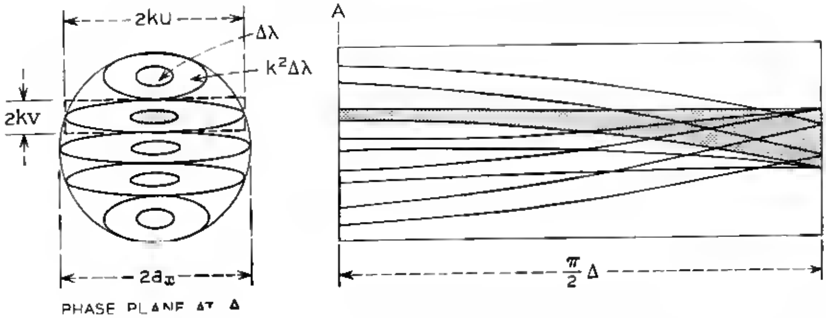


Fig. 4 — A possible distribution of phase spots in the useful phase area.

If the guiding medium is not homogeneous along the $z$ axis, but consists of a periodic sequence of lenses of width $2a_x$, the useful phase area may be different from that in Fig. 4. In this case, discrete apertures have to be considered at the positions of the lenses. (The intermediate guide diameter in a lens guide in general is immaterial, because between the lenses the beams have a smaller cross section and separation than at the lenses). Figure 5 shows the useful phase area and the spot pattern for confocally arranged lenses.

This case surmises that, proceeding from lens to lens, the rotation of the phase pattern is exactly 90°. Even if the tolerances for the focal lengths and lens spacings are very strict, these rotations will eventually, after many lenses, get out of step with respect to the lens positions and aperturing will occur when the phase pattern is rotated at any angle in the phase plane. This situation is shown in Fig. 6. If this happens, the useful phase plane is restricted to a circular area. It seems, therefore, that Fig. 4 represents a more general case for practical applications.

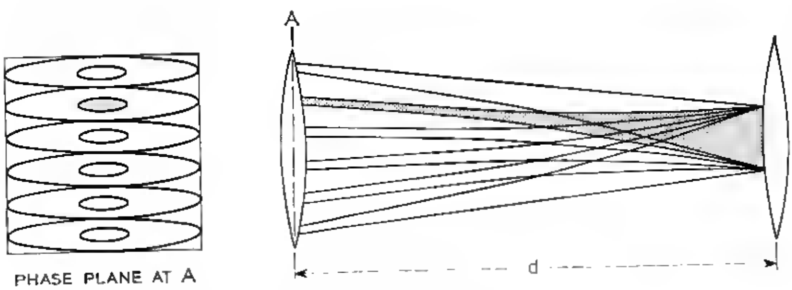


Fig. 5 — The useful phase area for a confocal imaging system.

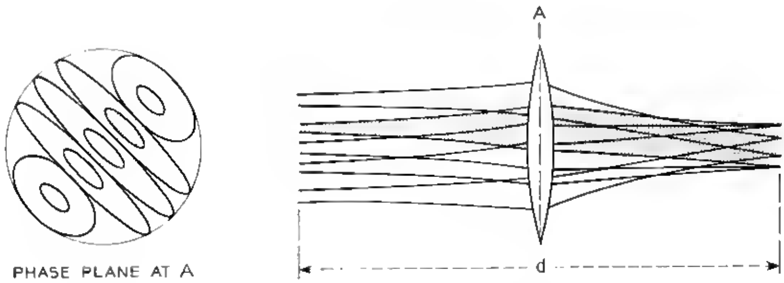


Fig. 6 — The phase pattern is out of step with the confocal lens position.

The number of phase spots that can be fitted into the circular area $\pi a_x^2$ of Fig. 4 is approximately

$$n = \frac{\pi a_x^2}{4k^2 uv} = \frac{\pi^2 a_x^2}{4k^2 \Delta\lambda}, \tag{7}$$

assuming that the area occupied by one spot may be approximated by the dotted rectangle in Fig. 4. For large numbers of beams ($n \geqq 10$), this approximation is satisfactory.

There is no reason why the beams have to be arranged the way they are in Fig. 4. There is no restriction on shape and location of the phase spots in the useful phase plane. Of course, arranged as in Fig. 4, the beams are clearly separated at distinct cross sections, which makes launching and receiving a simple and straightforward matter.

E. A. J. Marcatili of Bell Telephone Laboratories suggested the arrangement shown in Fig. 7 and demonstrated how such beams may be launched: a common lens is used for every overlapping group of beams feeding every member of the group at a different angle. At a distance $\pi\Delta/2$ from this lens, or at multiples of this distance, the

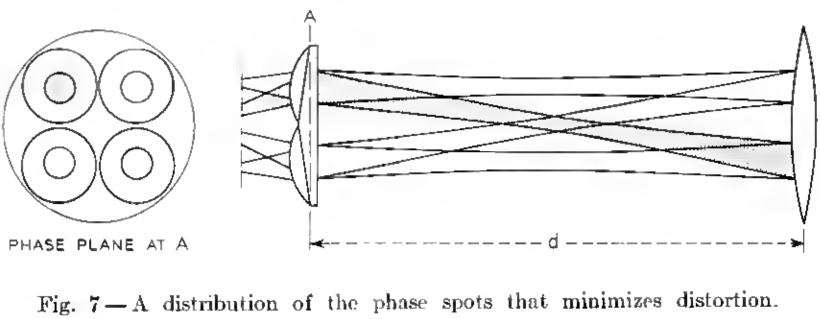


Fig. 7 — A distribution of the phase spots that minimizes distortion.

beams arrange themselves in groups again and could he separated hy means similar to the transmitter scheme.

Whereas the beams in Fig. 4 vary considerably in width as they propagate along the guide, the beams in Fig. 7 constantly keep the minimum width $w$ as given by (3). From the nature of the distortions on optical surfaces, it might be expected that large beams will he distorted more than small ones. In this case, the arrangement in Fig. 7 might be less susceptive to mutual interference of beams and consequently permit a smaller $k$ factor and a larger capacity. For equal $k$ there is no difference in capacity of hoth schemes, at least not within the accuracy of (7) which was derived for large $n$. Other arrangements as well as combinations of the schemes in Figs. 4 and 7 may be practical for certain cases.

By applying the phase space technique to three-dimensional systems, some of the lucidity is lost, but one can still gain some interesting results. If the beam waveguide has a cylindrical cross section of radius $A$, a circular area with the radius

$$a_y = (A^2 - a_x^2)^{\frac{1}{2}} \tag{8}$$

in the $y$-phase plane is available simultaneously with the area $\pi a_x^2$. The total useful phase space is consequently

$$S = \int_0^{\pi A^2} \pi a_y^2 \, d(\pi a_x^2). \tag{9}$$

By inserting (8) into (9), one has

$$S = \tfrac{1}{2}\pi^2 A^4. \tag{10}$$

Allowing rectangular areas for the phase spots in both the $x$- and $y$-phase plane, as in the two-dimensional example, the total capacity is found to be approximately

$$N = \frac{\frac{1}{2}\pi^2 A^4}{(4k^2\, \Delta\lambda/\pi)^2} = \frac{\pi^4}{32}\, \frac{A^4}{k^4 \Delta^2\, \lambda^2}. \tag{11}$$

If the total numher of heams is large ($n \geqq 100$), the rectangular approximation for the area occupied hy the phase spots is satisfactory and (11) holds independent of the way in which the beams are arranged in the guide. Figure 8 shows a nomogram hased on (11) for an optical wavelength of 1 micron. Given the radius, lens spacing, and filling factor $k$ of a lens guide, one can easily find the possihle capacity. Consider, for example, lenses spaced confocally hy 100 $m$. If their useful optical area

has a radius of 10 cm, and the filling factor is $k = 3$, approximately 300 beams could be transmitted in parallel.

Comparing (11) with M. von Laue's formula for the spatial degrees of freeedom of an optical system,[4] one finds that $N$ approaches the classical limit for $k$ about 1, that is, the beams would have to overlap at their $1/e$ amplitudes in order for the capacity of the guide to be fully used. There are many reasons why this limit cannot be reached in practice. Particularly important are the imperfections in the guide itself.

## IV. BEAMS IN CAVITIES

Tbe rules of gaussian beam geometry can also be used for optical cavities. Considering the cavity as a folded beam waveguide, possible beam paths can be traced using the phase plane. This way the useful capacity can be found for delay or storage applications.

Figure 9a shows a 2-dimensional square-law medium. The two plane surfaces $M_1$ and $M_2$ are highly reflecting mirrors and form an optical cavity. A beam, launched off-axis through the center hole of mirror $M_1$, would perform several round trips between the mirrors before hitting the entrance hole and leaving the cavity. Figure 9a un-

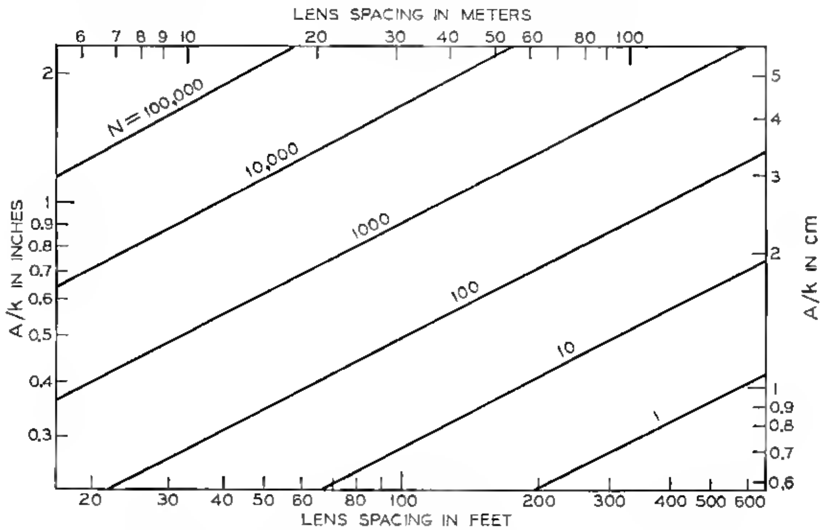


Fig. 8 — Nomogram evaluating the guide capacity for $\lambda = 1$ micron.

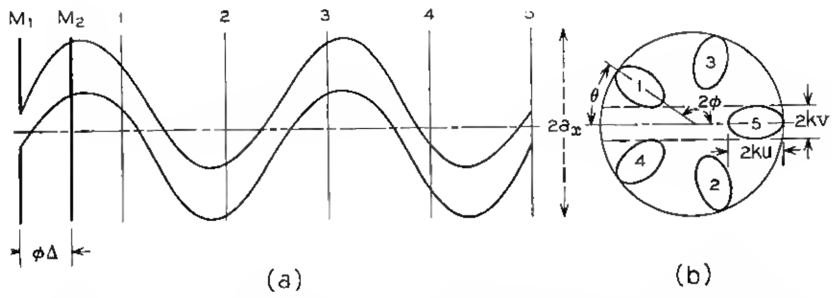


Fig. 9 — The beam path in a storage cavity consisting of a focusing medium between the mirrors $M_1$ and $M_2$. (a) The beam path unrolled along the axis. (b) The phase spots at the mirror $M_1$.

rolls the beam path along the guide axis. Figure 9b shows the phase spots for the cross sections 1 to 5 which correspond to reflections of the mirror $M_1$. In agreement with the previous considerations, the spots correspond to tubes $k$ times wider than the $1/e$-width of the beam. It is assumed that interference between different round trips and distortion is tolerable if the phase spots do not intersect one another, the boundary of the useful phase area, and the area occupied by the center hole (area between the broken lines in Fig. 9b).

Obviously, the total number of round trips can be increased by decreasing the angle $\theta$ shown in Fig. 9b. $\theta$ is smallest when the phase spots just touch the broken lines. Also there should be an optimum shape of the phase spots for which $\theta$ is a minimum. Though the area $k^2 \Delta\lambda$ of a phase spot is fixed, the main axes $u$ and $v$ can be chosen. Particularly if the cavity radius $a_x$ is large and a large number of round trips is to be stored in the cavity, the best ellipses will be long and thin, and the center hole diameter 2kv will be small.

It is now a simple matter of geometry to calculate the exact parameters. From the requirement that spot 1 touch the broken line, one finds

$$\sin \theta = \frac{2kv(a_x - ku)}{a_x^2 - 2a_x ku + k^2v^2}. \tag{12}$$

As indicated above, $v$ will be much smaller than $a_x$ and $u$ for optimum systems with large capacity. By neglecting $v^2$ in the denominator and replacing $v$ by (4) in the numerator, one has

$$\sin \theta \cong \frac{2\,\Delta\lambda\, k}{\pi a_x u} \frac{a_x - ku}{a_x - 2ku}. \tag{13}$$

The derivative $d \sin \theta/du$ vanishes for the optimum value

$$u_{\text{opt}} = \frac{a_x}{k}\left[1 - \frac{1}{(2)^{\frac{1}{2}}}\right] \tag{14}$$

and it turns out that

$$v_{\text{opt}} = \frac{\Delta\lambda\, k}{\pi a_x} \frac{1}{1 - 1/(2)^{\frac{1}{2}}} \tag{15}$$

is indeed small for large $a_x$. Under these conditions $\theta$ will also be small, and by replacing $\sin \theta$ by $\theta$ one finds

$$\theta_{\min} \cong \frac{\Delta\lambda\, k^2}{\pi a_x^2}\left[1 - \frac{1}{(2)^{\frac{1}{2}}}\right]^{-2}. \tag{16}$$

The maximum number of round trips is

$$n = \frac{2\pi}{2\theta_{\min}} = \frac{\pi^2 a_x^2}{\Delta\lambda\, k^2}\left[1 - \frac{1}{(2)^{\frac{1}{2}}}\right]^2. \tag{17}$$

For $n \geqq 10$, this formula gives satisfactory results.

The proper length of the cavity in Figure 8a is

$$\Phi\Delta = \tfrac{1}{2}(\pi - \theta_{\min})\Delta \tag{18}$$

with $\theta_{\min}$ from (16). If, instead of a homogeneously focusing medium, concave mirrors are used, (5) and (6) determine the mirror spacing $d$ and the focal length $f$. In connection with (17) and (18) one has

$$d = \Delta \cos \frac{\theta_{\min}}{2} \cong \Delta \tag{19}$$

and

$$1 - \frac{d}{2f} = \cos \Phi = \sin \frac{\theta_{\min}}{2} \cong \frac{\pi}{2n}. \tag{20}$$

Notice that for large $n$ the mirrors are almost confocally spaced.

Knowing the solution in the $x$ plane, one would like to solve the three-dimensional problem by just doing the same in the $y$ plane. Figure 10 shows equivalent phase planes for the $x$- and $y$-axes of the end mirror. Projection into the mirror plane yields the actual beam cross sections. Though it will be shown later that this arrangement is not quite optimum, Fig. 10 is very useful to calculate the cavity radius $A$ necessary to accommodate this or, later on, an improved beam path. The maximum displacement $a_x/(2)^{\frac{1}{2}} = a_y/(2)^{\frac{1}{2}}$ occurs simultaneously in the $x$ and $y$ directions. The total displacement of the beam

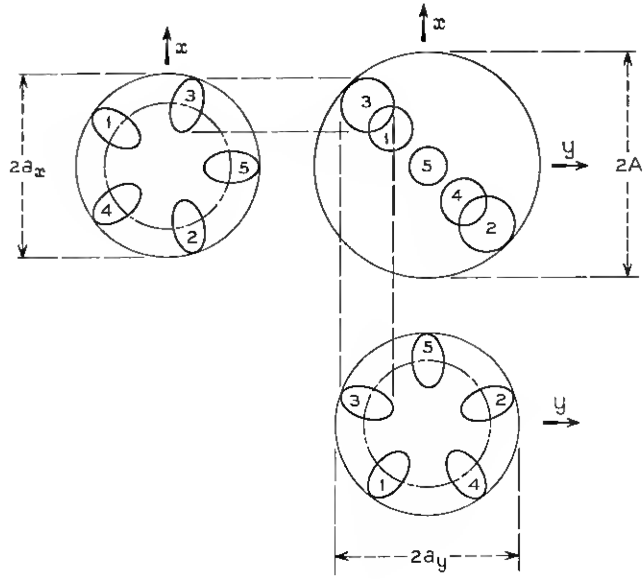


Fig. 10 — Construction of the beam cross sections at the mirror surface to determine the radius A of a cylindrical cavity.

axis is therefore $a_x = a_y$. The maximum displacement coincides with the largest beam cross section whose radius is $u_{\text{opt}}$ as given in (14). Consequently, a radius

$$A = a_x + ku_{\text{opt}} = a_x\left[2 - \frac{1}{(2)^{\frac{1}{2}}}\right] \tag{21}$$

is necessary for a cylindrical cavity.

The capacity can be increased drastically by deliberately introducing astigmatism as described in Ref. 1. Suppose the beam behaves in the $x$ plane as shown in Fig. 9 but, simultaneously, oscillates in the $y$ plane in such a way that it returns to tbe center of tbe $y$ plane already after 4 round trips. It can only leave tbe cavity when it is displaced neither in the $x$- nor the $y$-direction, and tbat happens for the first time after 20 round trips. Generally speaking, one acbieves

$$N = 2n(n - 1) \tag{22}$$

transits by this method. Technically, this can be done by warping one or the other of the mirrors slightly. Writing (20) for both $x$ and $y$ plane and subtracting one from the other yields, by using (22),

$$\frac{d}{f_x} - \frac{d}{f_y} = \frac{\pi}{n} - \frac{\pi}{(n-1)} = \frac{2\pi}{N}. \tag{23}$$

For large $n$ the approximation $N \simeq 2n^2$ can be made and by using (17) and (21) one has

$$N \cong \frac{2\pi^4 A^4}{\Delta^2\, \lambda^2 k^4} \frac{1}{[3 + (2)^{\frac{1}{2}}]^4}. \tag{24}$$

This is about a factor of 6 less than the number given in (11), that is, the described method does not fill the total available capacity. Notice that, in Fig. 9b, there is room for exactly one more set of spots between the used spots. This space would be filled by a beam that followed the same path as the described one, but in opposite direction. The additional capacity could be exploited by reflecting the existing beam back on itself. It can be shown that, in the three-dimensional scheme, there is space for an additional totally independent path in the cavity. Both paths could be linked by an outside mirror. By reflecting the two linked beams back into itself, the number of round trips could be quadrupled.

Without considering these sophistications, let us investigate what (24) means in terms of storage capacity. For large $N$ (19) can be used to calculate the total length of the beam path which, with the numerical factors evaluated, is

$$l = Nd = \frac{A^4}{2\, d\lambda^2 k^4}. \tag{25}$$

Surprisingly enough, this path is longest for a small cavity length $d$. Of course, the number of bounces (and consequently the losses) increase in a short cavity. The best mirrors available introduce a reflection loss of 0.05 percent or 43.5 dB attenuation after 20,000 bounces.[8] This corresponds to $10\mu s$ delay in a 15 cm cavity with mirrors 4 cm in diameter. If part of the loss is compensated by an amplifying material in the cavity, the number of round trips is eventually limited by scattering in the system.[9]

Perpetual recirculation of PCM information could he achieved by using an arrangement that amplifies $2\pi$-pulses[10] or a fast saturating absorber in combination with a suitable laser amplifier.[11] In both cases only pulses of a certain length and intensity are amplified, while any other signal is attenuated. It would be sufficient to provide the amplication at a few particular parts of the folded path where it is spatially separated from other round trips. If enough amplification

of this kind is provided to make up for all losses, pulses of the proper kind could circulate in the cavity perpetually without noise building up.

If the mirrors are moved so close to one another that they touch at the circumferences, the paraxial approximation, basis for the previous calculations, loses its validity. By extrapolating (25) into this range, however, one finds some interesting, though speculative, results. The center of the confocal mirrors are now spaced by $d = 2A/(3)^{1/2}$. For a bandwidth $b$ small compared to the light frequency $\nu$, the capacity is

$$c = \frac{bl}{\nu\lambda} = \frac{0.433}{k^4} \frac{b}{\nu} \frac{A^3}{\lambda^3} \tag{26}$$

with $l$ from (25).

It is easy to calculate the volume $V$ of this nut-shaped cavity. It is

$$V = \frac{10}{9(3)^{\frac{1}{2}}} \pi^2 A^3. \tag{27}$$

The number of the degrees of freedom of a cavity whose dimensions are large compared to $\lambda$ is independent of its shape and has the value*

$$c_{\mathrm{th}} = \frac{8bV}{\nu\lambda^3}. \tag{28}$$

In other words, the maximum number of bits which $V$ can hold in the form of electromagnetic energy is $c_{\mathrm{th}}$. By using (26), (27), and (28), one finds the (extrapolated) efficiency of the beamfolding method to be

$$\frac{c}{c_{\mathrm{th}}} = \frac{1}{(3.3k)^4}. \tag{29}$$

For $k = 3$, this efficiency is only $10^{-4}$, but even then a capacity of 16 k bit seems achievable with 1 GHz bandwidth in a cavity with the radius $A = 1$ cm.

## V. CONCLUSIONS

Various methods can be used to transmit a multitude of beams through a lens guide in such a way that all beams are clearly resolvable at the receiving end. The number of beams which can be transmitted is proportional to the square of the guide cross section and may be of the order of 300 for a guide of 10 cm radius with lenses

* See, for example, Ref. 4.

spaced confocally by 100 m. In this case, the centers of adjacent beams would be spaced by 6 beamwidths at particular cross sections in the guide.

Linking the available channels end to end in a cavity produces a delay line or storage device. At 1 micron wavelength a 10 $\mu$sec delay can be achieved in an optimized cavity, 15 cm long and 4 cm in diameter. The storage capacity is inversely proportional to the cavity length. Hence, the ultimate configuration would consist of two confocal mirrors with their circumferences touching. Extrapolating the paraxial theory to this situation yields a capacity of 16kbit for a bandwidth of 1 GHz if the nut-shaped cavity has a radius of only 1 cm.

ACKNOWLEDGMENTS

We are very thankful for stimulating discussions and fruitful suggestions by R. Kompfner and E. A. J. Marcatili.

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